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How big is a small earthquake? Challenges in determining microseismic magnitudes.

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Motivation

Earthquake magnitudes are surprisingly difficult to estimate accurately. This is especially so when evaluating small earthquakes, for example, those caused by human activities such as shale gas stimulation, CO₂ and waste water storage, and enhanced geothermal systems. Uncertainties are created by a range of issues including: which 'magnitude' scale is used; what type of instrument records the earthquake; how the instruments are deployed; and the heterogeneity of the Earth between the source and the receivers. Errors can be larger than an order of magnitude in scale. For very small earthquakes this is not usually of much concern. However, occasionally, larger earthquakes induced by human activity are felt at the surface. This has led to regulatory frameworks that require accurate assessment of earthquake magnitudes before they reach the point of being felt. Hence, to monitor and mitigate felt seismicity there is a need to calculate accurate earthquake magnitudes in real-time. Regulatory monitoring of induced seismicity is becoming a key issue in the successful development of reservoir projects that involve stimulation or storage. Here we discuss the challenges with implementing such reservoir monitoring, and provide a suggested monitoring strategy.

Traffic light systems (TLS)

'Traffic light systems' (TLS) (e.g., Bommer et al., 2006) are sometimes used to mitigate induced seismicity, whereby operations are paused, stopped or amended based on the

characteristics of the recorded seismicity. The purpose of a TLS is not to eliminate induced seismicity - all hydraulic fracturing is accompanied by very small magnitude seismic events (termed “microseismicity”) as the rock is fractured by the fluid pressure. The purpose of a mitigation scheme is usually to minimise the discomfort felt by the local public, and to eliminate the potential for damage to nearby buildings. In 2011, hydraulic fracture stimulation in the Preese Hall region of NE England led to a series of seismic events, the largest of which were felt at the surface (Clarke et al., 2014). This led to a moratorium on shale gas development in the UK and the implementation of a TLS for the UK. Figure 1 shows a comparison of TLS in operation in different parts of North America and the UK. The striking feature is the variability of the thresholds, reflecting public concern and variations in population density.

The UK has the strictest scheme of those highlighted in Figure 1: the TLS amber- and red-light limits are currently set at magnitudes of $M = 0.0$ and $M = 0.5$, respectively. If the amber light is exceeded during pumping, the operator can continue pumping, but must perform a well integrity check. If the red light is exceeded during pumping the operator must immediately suspend injection, reduce pressure and monitor seismicity for any further events. The focal location and mechanism should be determined to see whether the seismicity is natural or, if induced, whether it accords with the assumptions and expectations set out in a hydraulic fracture plan, which in the UK must be submitted by the operator in advance of any injection. If the magnitude and ground motion of an induced seismic event conform with the assumptions and predictions in the hydraulic fracture plan, injection operations can resume after an 18-hour pause, subject to any mitigation or other measures as part of the agreed plan.

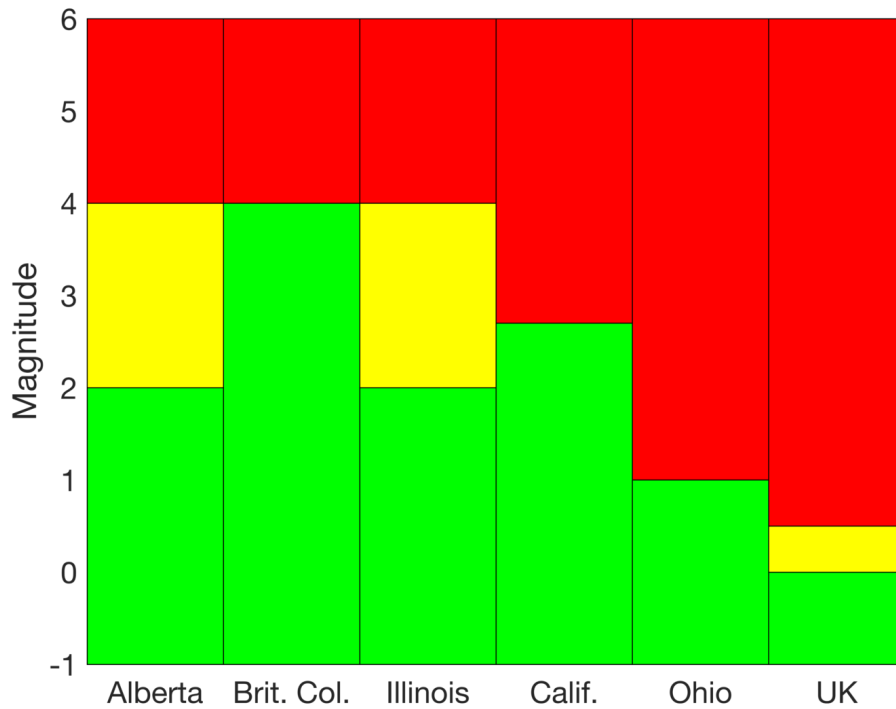


Figure 1. 'Traffic light system' thresholds as applied by regulatory jurisdictions in Canada, USA and the UK. The UK TLS has the lowest thresholds of any scheme used to regulate seismicity induced by hydraulic fracturing.

Rock failure and earthquakes – a brief review

Earthquakes are caused by a sudden release of stress when forces are enough to break the rock or overcome the friction along a pre-existing plane of weakness (a fault or fracture). The magnitude of this seismic event describes the energy it releases. Every year millions of small earthquakes occur naturally and go largely undetected – this includes those generated by human activity. Monitoring small earthquakes or microseismicity is now a routine tool for evaluating hydraulic fracture stimulation in tight-gas sandstones, shale gas formations and geothermal reservoirs. Most stimulated events are very small, but occasionally larger felt events are induced through stimulation and fault reactivation. For this reason, regulatory monitoring is a growing concern and is required for longer time periods and over larger areas.

Rock failure occurs when shear-stresses (σ_s) exceed the critical values described by the Mohr-Coulomb envelope,

$$\sigma_s = \tan \phi \sigma_n + C, \quad (1)$$

where (σ_n) is the normal stress, C is the cohesion, and ϕ is the internal friction. The latter two are intrinsic properties of the rock – stiffer rocks generally have higher coefficients of

cohesion and a steeper friction angles. Injection of fluids into the rock increase the pore pressure (P) and reduce the effective normal stress ($\sigma_n - P$). This means that earthquakes will occur at lower shear stresses. In the case of fault reactivation, a pre-existing weakness will have a lower cohesion or internal friction than the intact rock, again leading to rock failure at lower shear stresses. Faults may also be conduits for fluids, again reducing normal stresses.

Gutenberg and Richter (1944) showed that the magnitude-frequency relationship for an earthquake population follows a power-law behaviour,

$$\log_{10} N = a - bM, \quad (2)$$

where N is the number of events with a magnitude larger than M . This is the equation of a straight line where the intercept a -value is the number of events with $M \geq 0$ and describes the overall activity level within a region. The b -value is the slope of the line. A b -value of 1 implies that if there is 1 magnitude >3 event, there will be 10 magnitude >2 events, 100 magnitude >1 events, and so on. In global tectonic earthquake populations, b -values are commonly observed to be close to 1. However, b -values can vary depending on the stress regime and the role of fluids. Often during hydraulic fracture stimulation, where fluids reduce the shear strength required to rupture the rock, b -values are much higher than unity (>2 is not uncommon).

In the context of hydraulic fracture stimulation, high b -values are good, indicating fluid-triggered development of a fracture network, and low stress build-up. b -values near unity indicate higher stress build-up and possible fault reactivation. Accurate b -values require accurate measurements of magnitude across a range of scales.

The moment magnitude

There are a number of approaches to quantifying the size of a seismic event and thus far we have simply labelled the magnitude M . The moment magnitude (M_w) is the preferred scale as it is directly related to fault dimensions, the amount of slip of the fault and energy release. The seismic moment, M_o , is defined as,

$$M_o = \mu d A, \quad (3)$$

where μ is the shear modulus, d is the slip on the fault and A is the area of the fault that slips (see, *e.g.*, Aki and Richards, 2002). The moment magnitude is defined as (Hanks and Kanamori, 1979),

$$M_w = 2/3 (\log M_0 - 6.07), \quad (4)$$

where M_0 is measured in Nm. As these scales are logarithmic, each unit of magnitude increase relates to a ten-fold increase in displacement amplitude, which in turn corresponds to energy increasing by a factor of 32.

As we cannot directly measure the geometry of the fault and how it moves, we need to estimate the seismic moment through other methods. One approach employs the earthquake moment tensor, which is a mathematical representation of the magnitude and orientation of movement on a fault. However, this is time consuming to calculate and requires good station coverage. A more common approach for small magnitude seismic events is to evaluate the seismic moment from the moment function in the time or frequency domain (Figure 2) (*e.g.*, Stork et al., 2014). This signal is a product of the convolution of the fault rupture time and the rise time and its integral is proportional to the moment (a force times distance) of the earthquake rupture. The seismic moment, M_0 , in the frequency domain can be determined from the displacement pulse of a P - or S -wave arrival, and can be expressed as,

$$M_0 = 4\pi\rho v^3 r \Omega_0 / R, \quad (5)$$

where ρ is the density, v is the seismic velocity at the source (of the P - or S -wave, as appropriate), r is the source-receiver distance, and R is a radiation pattern correction. The term Ω_0 is the low-frequency level of the amplitude spectrum and is equal to the area under the displacement pulse (Figure 2).

Ω_0 can be estimated from amplitude spectra that are corrected for attenuation effects. At high frequencies the amplitude of the signal decays linearly and the point at which this decay starts is known as the corner frequency, f_c . As attenuation, Q , is generally unknown, determining Ω_0 normally involves fitting for Ω_0 , f_c and Q simultaneously using a

Brune source model (see Stork et al. (2014) for details). Furthermore, when dealing with small events (*i.e.*, microseismic events, where $f > 10$ Hz), the near-surface attenuates high-frequency energy making it difficult to estimate f_c (Anderson and Hough (1984)). The parameter, κ_0 , accounts for high-frequency energy decay beyond a site-dependent maximum frequency, f_{max} . The so-called kappa-corrected Brune model is,

$$\Omega(f) = \frac{\Omega_0 e^{-(\pi f r / v Q)}}{[1 + (f/f_c)^2]} e^{(-\pi f \kappa_0)}, \quad (6)$$

which is based on the traditional Brune source model with an additional exponential term involving κ_0 (Batlay and Hanks, 2014). This term is site dependent and there are range of approaches to estimate κ_0 (see, *e.g.*, Ktenidou, et al., 2014).

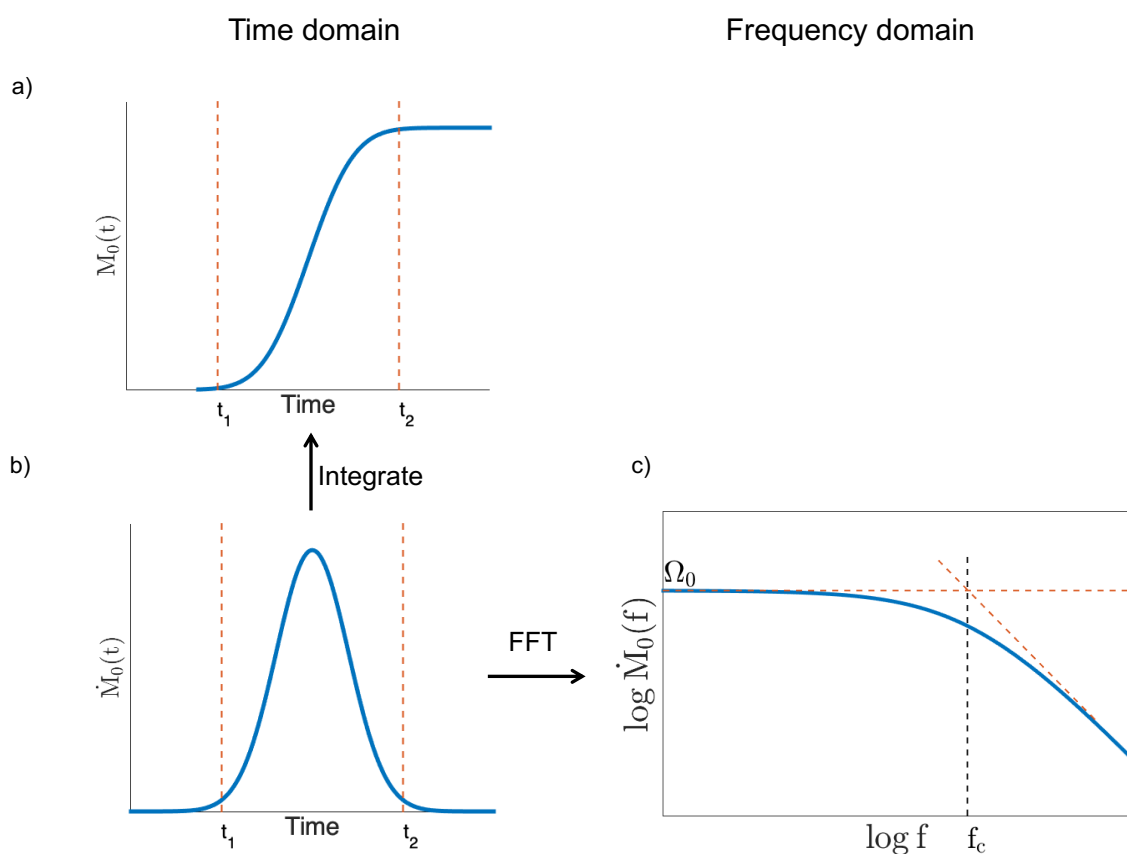


Figure 2. Earthquake model in the time and frequency domain. (a) The moment time function, $M_0(t)$, is the change in moment due to earthquake slip along a fault. This increases over the duration of the event, between times t_1 and t_2 . (b) The seismic moment rate

function, $\dot{M}_0(t)$, is proportional to the far-field displacement pulse (Aki and Richards 2002). (c) In the frequency domain, the moment rate spectrum is the Fourier transform of $\dot{M}_0(t)$. At low frequencies the spectral amplitude becomes constant (Ω_0) and is proportional to seismic moment. f_c is the corner frequency and FFT is the fast Fourier transform.

A number of factors can lead to significant errors in the estimation of M_w and, without care, variations of a unit in magnitude can be produced (Stork et al., 2014). The most significant errors can be caused by a failure to calculate the radiation pattern for the earthquake correctly (often a time-consuming exercise). This requires good station coverage and it is recommended that at least 4 stations with good azimuthal coverage are used. The calculations should be made for both the P - and S -wave phases. A correction for free-surface effects should also be included for stations deployed on the surface. Sampling rates need to be high enough to capture the corner frequency, f_c , which for $M_w \leq 0$ requires sampler rates > 1000 Hz. Another issue concerns the nature of the seismic sensor. A broadband instrument is required to capture the low-frequency signal needed to accurately record Ω_0 (Baig et al., 2010). Finally, a Q -corrected amplitude spectra should also be used and a correction for κ_0 should be applied if possible. Finally, good knowledge of the seismic velocity at the source is important, as it is cubed in equation (5), and it also affects the accuracy of the event location.

Estimating the moment magnitude, M_w , is easy in principle, but can be time consuming and requires a good signal to noise ratio. As most hydraulic stimulation stages generate thousands of earthquakes, we need a quicker method to estimate the source magnitude. The easiest approach is to use the largest observed amplitude of displacement recorded on a seismometer. This is the basis of the local magnitude, which follows the form of the original magnitude scale proposed by Richter (1935).

The local magnitude

The idea of local magnitude (M_L) was developed by Charles Richter in 1935, and is an empirical scale based on the ground displacement recorded on a particular type of instrument in southern California (Richter, 1935). M_L was a scale based on a $M=3$ representing 1mm of displacement on the horizontal-component of a Wood–Anderson seismometer at a distance of 100 km from the epicentre. The general form of a local magnitude scale is,

$$\begin{aligned}
M_L &= \log (A) - \log (A_o), \\
&= \log (A) - (a \log (r) + b(r) + c),
\end{aligned} \tag{7}$$

where r is the hypocentral distance. The term $(\log (A_o))$ is a displacement correction, which is comprised of three terms with coefficients a , b and c representing the corrections that need to be applied for geometrical spreading, attenuation and a base level, respectively. A is the zero-to-peak displacement amplitude measured by a seismometer (Figure 3), where its response has been converted to that of a short-period Wood–Anderson seismometer, an instrument that is no longer in routine use. This correction is broadly comparable to a 2Hz high-pass filter and a gain correction of 2080.

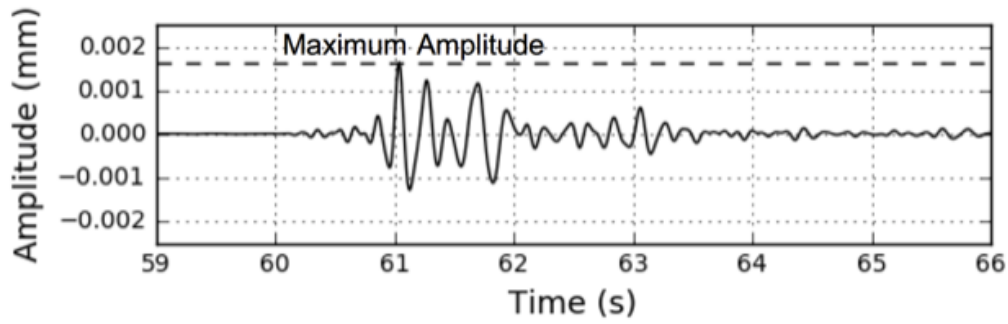


Figure 3. Example of a seismic event where the maximum amplitude of the S-wave used to determine M_L .

The analysis of amplitudes from earthquakes in a given region (*e.g.*, the UK) recorded by stations at a range of epicentral distances can be inverted to determine local values for the terms in the equation (7). For example, based on nearly 1500 observations from 85 earthquakes recorded by 50 stations, Ottemöller and Sargeant (2013) determined the following local magnitude scale for the UK,

$$M_L = \log (A) + 0.95 \log (r) + 0.00183(r) - 1.76. \tag{8}$$

The nominal detection level for the UK network is $M_L = 2.0$ and most stations are more than 50 km from any given earthquake. Furthermore, most earthquakes are tectonic in nature

and lie in the middle crust (>10 km in depth), so the recorded seismic waves have travelled primarily through deeper, crystalline crustal rocks (Figure 4).

In most industrial applications of earthquake monitoring (*e.g.*, shale gas stimulation), seismometers are deployed very close to the site (<10 km) and the earthquakes are shallow (<4 km) (Figure 4). As a result, seismic waves travel primarily through the low-velocity and highly-attenuating near-surface. This leads to a systematic overestimation of seismic local magnitudes, as the largest waveform now relates to a different phase (*e.g.*, a direct wave rather than a P_g phase in the deeper crust) than that used to originally derive the scale (Butcher et al., 2017). Figure 5 shows the recorded displacement amplitudes for two coal mining-induced events in NE England (described by Verdon et al., 2017) recorded by 6 stations in a local seismic array (<5 km) and 8 stations from the regional UK array that are >50 km from the events. On the distant stations, the displacements match well with the UK scale for an M_L 1.0 event. On the nearby stations, displacements are substantially larger, and this discrepancy increases as the hypocentral distance decreases.

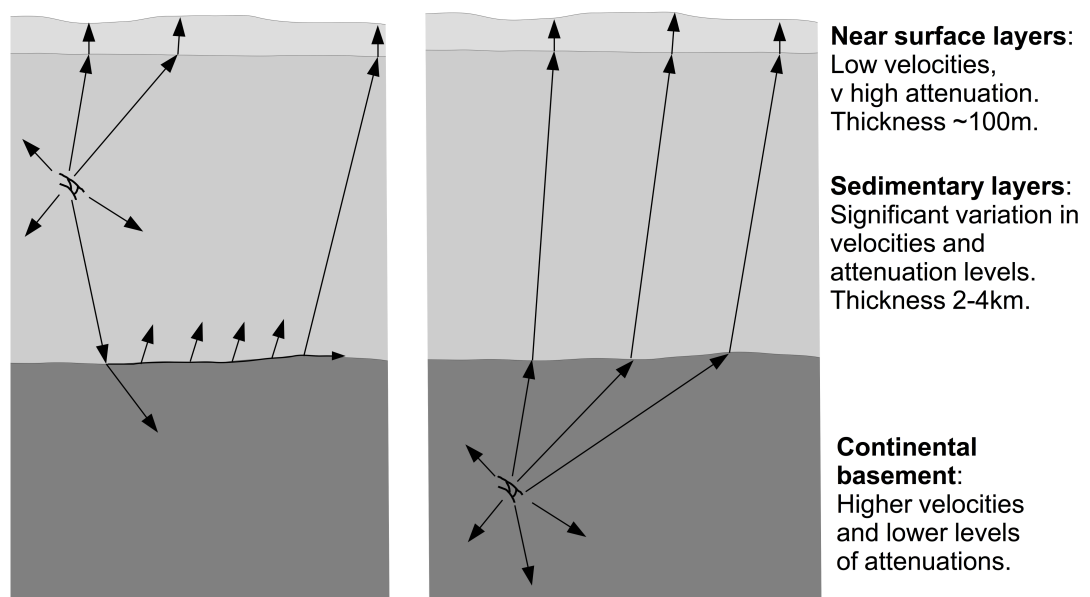


Figure 4. Schematic travel paths for: (left) shallow earthquake occurring in sedimentary layers; (right) deeper regional tectonic event.

To address this limitation a modified local magnitude scale is required for stations less than a critical distance. In this case that distance was selected at 17 km, but this is dependent on the local geology and will vary from region to region. Based on the analysis of

a cluster of similar coal-mining-induced seismic events, Butcher et al. (2017) develop a modified UK local magnitude scale for events in northern England where the source-receiver distance is less than 17 km,

$$M_L = \log (A) + 1.17 \log (r) + 0.0514(r) - 3.0. \quad (9)$$

Hence two M_L scales should be used – the UK scale (8) when the source-receiver separation is greater than 17 km, and the Butcher scale (9) when stations are < 17 km from the source. The biggest difference between equations (9) and (8) is in the attenuation terms, which are over an order of magnitude different in size (0.0514 versus 0.00183). More recently, Lockett et al. (2019) have combined the two scales using an exponential term to weight the near and far offset contributions, as adopting an entirely new scale would require the BGS to recalculate and republish its entire catalogue. This scale is expressed by

$$M_L = \log (A) + 1.11 \log (r) + 0.00189(r) - 2.09 - 1.16e^{-0.2r}, \quad (10)$$

which is the same as the scale used to estimate M_L in the UK, but corrects for the overestimation of magnitudes at near offsets. Recently this has been adopted for all UK M_L calculations.

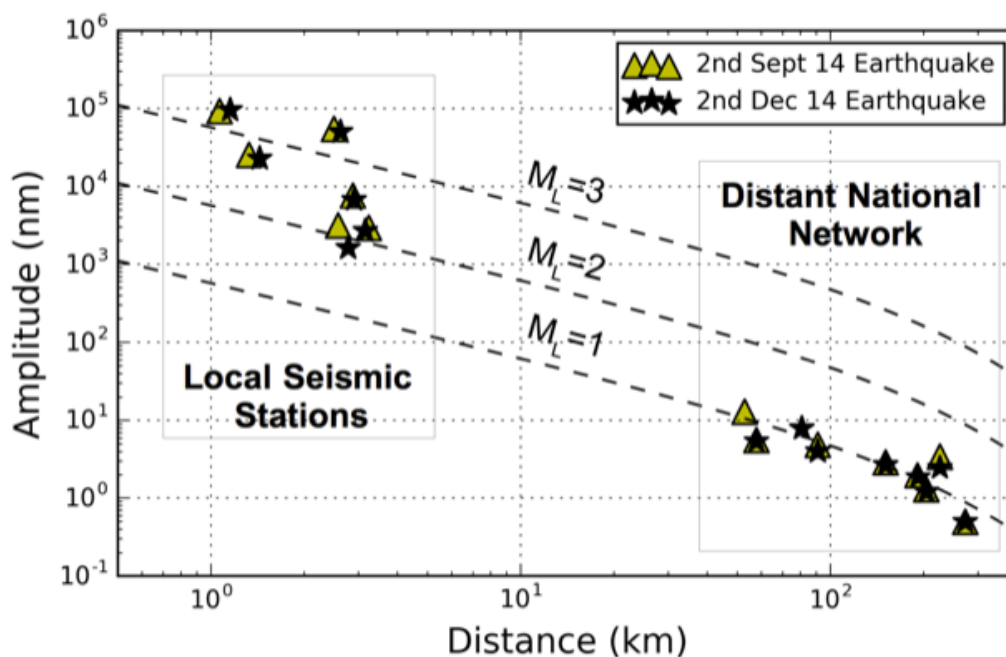


Figure 5. Recorded displacement amplitude versus distance for two coal-mining related earthquakes, as shown by stars and triangles, recorded by 14 stations. The solid, dashed and dotted lines show the corresponding magnitudes based on the UK scale for M_L 1.0, 2.0, and 3.0 (*i.e.*, using equation (7)). Amplitudes recorded on stations <5 km from the epicentre systematically estimate much larger magnitudes than those recorded by stations >50 km from the epicentre. See Butcher et al. (2017) for more detail.

Finding the right moment: M_w versus M_L

The moment magnitude scale was developed to address the well-known deficiency that local magnitude scales break down at large magnitudes. For example, the devastating 2011 Tohoku earthquake had a moment magnitude of M_w 9.1, but early estimates of the local magnitude were $\sim M_L$ 7.9. This is explained by saturation issues, where both the amplitudes and the frequency range are beyond the sensitivity of the instrument. Large earthquakes typically generate the majority of their energy at low frequencies (*i.e.* <1Hz), which is removed when estimating M_L , mainly due to the correction to a shorter-period Wood-Anderson response (Figure 6).

As previously discussed, M_L scales break down at small magnitudes, as microseismic events rarely contribute to the derivation of these empirical scales. This is partially because there was little historical interest in capturing these types of earthquakes, as a result of difficulties in recording them. In his 1958 textbook, Richter states that "comparatively little can be accomplished with seismograms belonging to the local earthquake range of distance", as instrument timing of this period could not accurately record these higher frequency events. Although this has been overcome with modern day instruments, they are still challenging to record, as to achieve good signal-to-noise ratios requires receivers located at distances typically <10km from the source. As a result, complete microseismic catalogues are rare and often only cover a very localized region.

Even when they are well recorded, local magnitudes are regularly observed to be smaller than moment magnitudes for magnitudes less than 3, with this discrepancy increasing as size of the event decreases. The primary cause of this discrepancy is due to a preferential decay of high-frequency energy as it propagates through highly attenuating layers (Deichmann, 2017), such as the near-surface (Figure 4). These act to reduce the maximum amplitude in the time-domain. Magnitudes calculated using M_L will therefore be smaller than M_w . M_w is estimated using the low frequency plateau, Ω_0 , in equation (5), and is therefore unaffected by the loss of high frequencies (Figure 6).

M_W is preferred by seismologists, as it is directly related to the seismic moment (M_0), which is the best measure of earthquake size. Nevertheless, it can be difficult to estimate accurate moment magnitudes with the low signal to noise ratios characteristic of these small events. It is therefore desirable to apply a correction to local magnitudes to infer moment M_W magnitudes and the seismic moment, M_0 . A comparison of local and moment magnitudes can be used to establish an M_L correction for a given region (e.g., Munafò, et. al. 2016).

As shown in equation (3), M_0 is related the amount of slip on a fault plane. However, there is an ambiguity in that large slip on a small fault plane is equivalent to small slip on a large plane. This is important, as the stress drop is different in each scenario. To better establish the fault area (and hence slip) and stress drop requires not only knowledge of the seismic moment (M_0), but also the corner frequency (f_c). As noted, attenuation in near-surface layers imposes a maximum frequency limit, f_{max} , leading to errors in estimates of f_c (see Figure 6), and hence errors in estimates of fault dimension and stress drop. These errors are most significant at seismic station close to an earthquake, and can produce differences in stress drops of up to two orders of magnitude. As mentioned, an estimate of κ_0 is a useful approach to obtain accurate corner frequencies, f_c , and numerous different approaches for estimated κ_0 are reviewed by Ktenidou, et. al. (2014). One such approach that is being currently explored is to use the ambient noise field to estimate the decay of high frequencies, which can be measured prior to operations that may result in microseismicity. This can allow for the development of physics-based relationships between M_L and M_W , and more accurate estimation of the properties of an earthquake.

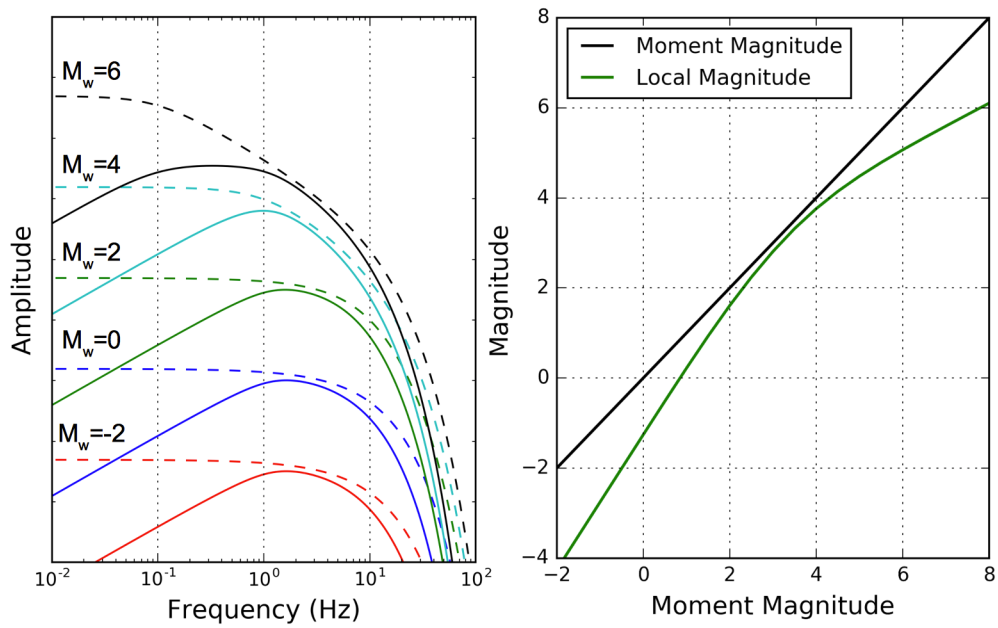


Figure 6. (Left) Dashed lines show the modelled frequency content of earthquake sources varying in moment magnitude from -2 to 6. The solid lines show the frequency content after including a Wood-Anderson instrument response and including the effects of near-surface attenuation on the high frequencies. (Right) Schematic figure showing differences between M_L and M_w , as derived from the source spectra on the left. Note that M_L is derived from the spectra shown in solid lines, and M_w is from those in dashed lines. M_L underestimates magnitudes when they are large and small.

Conclusions

The increasing use of Traffic Light Systems (TLS) to regulate induced seismicity in a range of settings requires accurate estimates of earthquake magnitudes in real time. However, there are a number of factors that lead to errors in estimates of seismic magnitudes. The use of the moment magnitude (M_w) is preferable, as this scale leads to an assessment of fault dimensions and stress release. However, in practice, local magnitudes (M_L) are simpler to use, especially in real-time applications. Measurements of microseismicity using either scale are affected by the local geology and near-surface attenuation. Getting things right involves using appropriate instruments, well placed sensor arrays and a good understanding of the near surface.

Accurate determination of the seismic moment (M_o) based on spectral estimates requires instruments with a broad-band response, a high (>1000 Hz) sample rate and a good signal-to-noise ratio. The spectra should be rich in low-frequency signal and a correction for

Q , and κ_0 if possible, is important. Estimates should be made from both P - and S -wave phases, using 4 or more stations deployed with good azimuthal coverage of source radiation patterns. A good knowledge of uncertainties in the velocity model and event locations should also be taken into account.

The use of local magnitude scales (M_L) requires careful calibration, as the scales will vary in form from region to region. Furthermore, signals recorded near the source will have a tendency to overestimate magnitudes in comparison with signals recorded at stations with epicentral distances more than 15-20 km. This is because raypaths to the near-offset stations travel through more highly-attenuating near-surface sedimentary layers. Raypaths to more distant stations are primarily through less-attenuative basement rocks. A separate magnitude scale must be therefore derived for near-offset stations, which requires knowledge of baseline natural- and/or induced-seismicity in a region.

Moment and local magnitudes diverge with large earthquakes ($> M 6.0$), but also with microseismic earthquakes ($< M 2.0$). The former is due to saturation issues, as M_L is calculated using a Wood-Anderson instrument response, with frequencies lost below $\sim 2\text{Hz}$. M_w should always be used for large events. The latter relates to the attenuation affecting high frequencies ($> 10\text{Hz}$). A regional comparison of M_L and M_w can be used to develop a correction. Furthermore, a correction for the exponential loss of high-frequency energy is required to ensure accurate estimates of corner frequencies and earthquake parameters such as fault dimensions, fault slip and stress drop.

In conclusion, monitoring arrays that address operator needs (*e.g.*, low detection thresholds) will be very different from arrays required for regulatory purposes (*e.g.*, accurate magnitudes). As induced seismicity continues to attract public attention, the next few years will see the rapid development of monitoring strategies for regulatory purposes.

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